PHYSICS 5B MT Crib Sheet - DIVIT RAWAL

MAXWELL'S EQNS

$\nabla \cdot \vec{E} = \rho/\epsilon_0$ $\quad \nabla \cdot \vec{B} = 0$

$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ $\quad \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

$\rho = q/\text{vol}$ $\quad J = I/\text{area}$

Lorentz Force Law

$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Charge & Energy conserved

MATH

Divergence: $\nabla \cdot \vec{F}$

Curl : $\nabla \times \vec{F}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Flux & Gauss' Law

$\vec{E}$ is conservative

$$\Phi_E = \oint_S \vec{E}(\vec{r}) \cdot d\vec{a} = \int_V d^3r\, \underbrace{\nabla \cdot \vec{E}(\vec{r})}_{\text{flux/vol at } \vec{r}} = \int_V d^3r \frac{\rho(\vec{r})}{\epsilon_0} = \frac{Q_{enc}}{\epsilon_0}$$

Lines of constant φ are locally perpendicular to $\vec{E}$ (equipotential)

Coulomb's Law & Superposition

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \hat{r}_{21}$$

$$\nabla(\vec{E}_1 + \vec{E}_2) = \frac{\rho_1 + \rho_2}{\epsilon_0} = \nabla \vec{E}_1 + \nabla \vec{E}_2$$

$$\Rightarrow E(\vec{r}) = \sum_{n=1}^{N} \frac{q_n (\vec{r} - \vec{r}_n)}{|\vec{r} - \vec{r}_n|^3} = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(r')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Work-Energy Thm.

from a → b

$$W_{ab} = K_b - K_a = q \int_{\vec{r}_a}^{\vec{r}_b} \vec{E}(\vec{r}) \cdot d\vec{r} = \int_{\vec{r}_a}^{\vec{r}_b} \vec{F}(\vec{r}) \cdot d\vec{r}$$

$$dW_{ab}^{ext} = -q\vec{E}(\vec{r}) \cdot d\vec{r} \Rightarrow d\varphi(\vec{r}) = \frac{dW^{ext}}{q} = -\vec{E}(\vec{r}) \cdot d\vec{r}$$

$$\Rightarrow W_{ab}^{ext} = q[\varphi(\vec{r}_b) - \varphi(\vec{r}_a)] = K_a - K_b \text{ s.t. } \vec{E}(\vec{r}) = -\nabla \cdot \varphi(\vec{r})$$

Taking $\varphi(\infty) = 0$ as ref, $\varphi(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}$ for a pt. charge

Conductors & Capacitance

If no charges in an equipotential surface, $E = 0$ everywhere inside

Equilibrium $\Rightarrow E(s) \propto \hat{n}(s)$

Surface of conductor is equipot.

Statements about conductors

(i) charges move to surface

(ii) $\vec{E}(s) \propto \hat{n}(s)$ & S is an equipotential

(iii) $\rho = 0$ inside (iv) $\vec{E} = 0$ inside

(v) $\vec{E}(\vec{r}) = \frac{\sigma(\vec{r})}{\epsilon_0}$ outside

$C = \frac{Q}{\varphi_1 - \varphi_2} = \frac{Q}{V}$

For ‖-plate cap. $C = \frac{\epsilon_0 A}{d}$

Work to charge capacitor: $W = \frac{CV_0^2}{2}$

Energy stored in E-field: $\frac{\epsilon_0 E^2}{2} = U$ (per vol)

Conductivity thru $\vec{J} = \sigma \vec{E}$

$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ $\quad I = V/R$

Force on current-carrying conductor in magnetic field $F = I\vec{L} \times \vec{B}$

$\vec{L}$ length vector of ~~current~~ conductor w/in mag. field

$U_E = \frac{\epsilon_0}{2} \int_V E^2 dV$

$U_B = \frac{1}{2\mu_0} \int_V B^2 dV$

Poisson's Eqn.

$$\nabla^2 \varphi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

$$\Downarrow$$

$$\varphi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

Laplace's Eqn: $\nabla^2 \varphi(\vec{r}) = 0$

a soln. to Laplace's Eqn. is unique

If $\varphi_1(\vec{r})$ is soln. to $\rho_1(\vec{r})$ & $\varphi_2(\vec{r})$ is soln. to $\rho_2(\vec{r})$, then $\alpha\varphi_1(\vec{r}) + \beta\varphi_2(\vec{r})$ is soln. to $\alpha\rho_1(\vec{r}) + \beta\rho_2(\vec{r})$

Biot-Savart Law

dir by RHR

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint d\vec{l} \times \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

Ampere's Law

$$\int_S \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Faraday's Law

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$I = \frac{\mathcal{E}}{R}$$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ — area vec. perp. to current loop

$|a \cdot b| = ab\cos\theta$

$|a \times b| = ab\sin\theta$

Motional EMF

$\mathcal{E} = \vec{v} \times \vec{B} \cdot \vec{L}$

$= vBL\sin\theta$

Lenz's Law: Induced current flows in a direction that opposes change in magnetic flux thru the loop

Motion in E-field

$\vec{a} = \frac{q\vec{E}}{m}$ $\quad x = \frac{1}{2}\frac{qE}{m}t^2$

$v = \frac{qE}{m}t$

Coords

Cart → Cylr: $r = \sqrt{x^2 + y^2}$, $\varphi = \tan^{-1}\left(\frac{y}{x}\right)$, $z = z$

Cyl → Cart.: $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

Cart → Spher.: $r = \sqrt{x^2 + y^2 + z^2}$

from z → $\theta = \cos^{-1}\left(\frac{z}{r}\right) = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

in xy → $\varphi = \tan^{-1}(y/x)$

Sphere → Cart: $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$

$\det(J_{cart \to cyl}) = r$

$\det(J_{cart \to sphere}) = r^2\sin\theta$

Magnetic Field from a wire: $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$

Magnetic Field in a solenoid: $B = \mu_0 n I$ ← turn density N/L

Magnetic Field inside a Toroid:

$$B = \frac{\mu_0 N I}{2\pi r}$$

$V_{sphere} = \frac{4}{3}\pi r^3$ $\quad$ $A_{circle} = \pi r^2$

$SA_{sphere} = 4\pi r^2$ $\quad$ $Circ_{circle} = 2\pi r$

Common Scenarios

Infinite Line of charge $E = \frac{\lambda}{2\pi\epsilon_0 r}$

Inf. Sheet of Charge: $E = \frac{\sigma}{2\epsilon_0}$

Vector Calc Identities:

$\nabla \times (\nabla f) = 0$

$\nabla \cdot (\nabla \times A) = 0$ $\quad$ $k = \frac{2\pi}{\lambda}$

$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$

Plane Waves

$\vec{E} \perp \vec{B} \perp$ dir. prop. $\quad |\vec{E}| = c|\vec{B}|$

$\vec{E}(\vec{r},t) = \hat{x} E_0 \operatorname{Re}\left[(e^{ikz} + e^{-ikz}) e^{-i\omega t}\right]$ $\quad$ $\vec{B}(\vec{r},t) = \frac{k\hat{y}}{\omega} \operatorname{Re}\left[E(e^{ikz} - e^{-ikz}) e^{-i\omega t}\right]$

$\vec{E}(\vec{r},t) = 2\hat{x} E_0 \cos kz \cos \omega t$ (standing) $\quad$ $\vec{B}(\vec{r},t) = 2\frac{k}{\omega} E_0 \hat{y} \sin kz \sin \omega t$

$\cos(kz - \omega t)$ (travelling)

$U \equiv$ energy/vol $\quad U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\epsilon_0\left(E_0^2 + \frac{B^2}{\mu_0\epsilon_0}\right)\cos^2(kz-\omega t) = \epsilon_0 E_0^2 \cos^2(kz - \omega t)$

$\vec{S}$ = [energy density] × [velocity] $\Rightarrow |\vec{S}| = \epsilon_0 c E^2 \cos^2(kz-\omega t)$ $\quad |\vec{S}| = \frac{\text{power}}{\text{area}}$

$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ $\quad \langle S \rangle = \frac{1}{Z_0}\frac{E_0^2}{2}$

$c = \omega / k$ $\quad I = \frac{1}{2} c\epsilon_0 E^2$

Dielectrics

$dQ(s) = \vec{J}_b(s)\, dt \cdot d\vec{a}$ $\quad$ $\sigma_b(s,t) = \frac{dQ(s,t)}{da} = \int_0^t \vec{J}_b(s,t')\,dt' \cdot \hat{n}(s)$

$\rho = \rho_f + \rho_b$ $\quad \nabla \cdot E = \rho/\epsilon_0$

$\sigma_b = \vec{P} \cdot \hat{n} \Rightarrow \frac{\partial \vec{P}(s,t)}{\partial t} = J_b(s,t)$

$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$

$\rho_b = -\nabla \cdot \vec{P}$

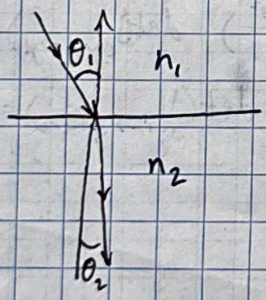
$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$ s.t. $\nabla \cdot \vec{D} = \rho_f$

For linear dielectric

$\vec{P} = \epsilon_0 \chi \vec{E} \Rightarrow \vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$ $\quad K \equiv 1 + \chi$ s.t. $\vec{D} = \epsilon_0 K \vec{E}$ & $\epsilon = \epsilon_0 K$

$C = K C_{vac}$, $Q = K Q_{vac}$ if V const. $\quad \nabla^2 \vec{E} = \mu_0 \frac{\partial^2}{\partial t^2}\vec{D}$

$\frac{\omega}{k} = \sqrt{\frac{1}{\mu_0 \epsilon}}$ = speed of light in dielectric $\quad B = \frac{n}{c}E$ $\quad n(\omega) = \sqrt{\frac{\epsilon_0}{\epsilon(\omega)}}$ $\quad K(\omega) = \frac{n(\omega)}{c}\omega$

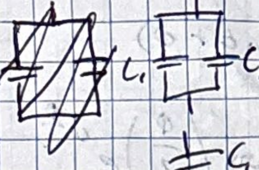


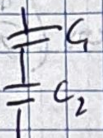
Snell's Law

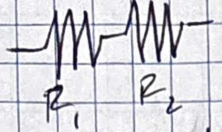
$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$ $\quad R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$ $\quad T = 1 - R$

← incident from denser medium

$\theta_c = \sin^{-1}\frac{n_1}{n_2}$ $\quad$ for $\theta_i > \theta_c$, $R = 1$ & $T = 0$

 C_1 C_2 $C_{eq} = C_1 + C_2$

 C_1 C_2 $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

 R_1 R_2 $R_{eq} = R_1 + R_2$

 $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

More Dielectrics

$\vec{P}$ from $-$ to $+$ $\quad$ $\vec{D}$ works w/ Gauss' Law

$K = 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$ $\quad$ $\vec{E} = \frac{1}{\epsilon_0}(\vec{D} - \vec{P})$

Usually, $\vec{P} = \epsilon_0 \chi_e \vec{E}$

$\int_V \nabla \cdot \vec{D} = \oint_S \vec{D} = Q_{free,\, enc}$

More Wave Stuff

$S = \frac{E \times B}{\mu_0}$ $\quad$ $\langle S \rangle = \frac{1}{2\mu_0} \mathrm{Re}\left[E \times B^*\right]$

$U = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$ $\quad$ $S = \frac{E_0^2}{\mu_0 c} \hat{k}$

$S_{inc} = S_r + S_t$

$R = \frac{|S_r|}{|S_{inc}|}$ $\quad$ $T = \frac{|S_t|}{|S_{inc}|}$ $\quad$ $Z_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$ $\quad$ $Z_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$ μ_0 $\quad$ $Z = \frac{c}{n}$

$\Rightarrow R = \left|\frac{Z_2 - Z_1}{Z_2 + Z_1}\right|^2$, $T = 1 - R$

TIR only occurs dense $\to$ not as dense

$\theta_i = \theta_r$

$\sin\theta_c = \frac{n_2}{n_1}$

denser

Conductor Stuff

conductivity

$\sigma = \frac{1}{\rho}$ $\leftarrow$ resistivity

$J = \sigma E$

$I = \int J \, dA = \sigma E A$

$\boxed{E \text{ inside conductor} = 0}$

No net polarization in conductor @ eq.

X-Prod RHR

Parallel RHR

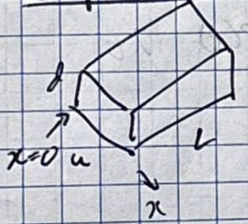
Thumb in $\vec{A}$

rh fig.

$A \times B \to C$

curl in dir $\quad$ thumb

Capacitor Prac Prob



$\epsilon(x) = \epsilon_1 + \frac{x}{w}\epsilon_2$ $\quad$ Charge to V.

(a) determine $\rho_f(x)$

(b) Capacitance of device

(a) $\frac{Q_{enc}}{A\epsilon_0} = \vec{E} \Rightarrow Q_{enc} = CV \cdot \frac{A}{Lw} \Rightarrow \vec{E} = \frac{CV}{\epsilon_0 L w}\hat{z}$

$\Rightarrow D = \left(\epsilon_1 + \frac{x}{w}\epsilon_2\right)\frac{CV}{\epsilon_0 L w} \Rightarrow \vec{\nabla} \cdot \vec{D} = 0 = \rho_f$

(b) Consider superposition of many capacitors in parallel

$C_{one\ strip}: \frac{k\epsilon_0 A}{d} = \frac{\left(\epsilon_1 + \frac{x}{w}\epsilon_2\right)\epsilon_0\, dx\, L}{d}$

$\Rightarrow \int_0^w \frac{\epsilon_0 L\left(\epsilon_1 + \frac{x}{w}\epsilon_2\right)}{d}\,dx \Rightarrow \frac{\epsilon_0 L}{d}\left(\epsilon_1 w + \frac{w}{2}\epsilon_2\right)$

Wave Problem Ex.

Take $E_1 = E_0 \hat{e} e^{i(kx-\omega t)}$; $E_2 = E_0 \hat{e} e^{i(kx-\omega t+\phi)}$; $k = \frac{2\pi}{\lambda}$

(a) $E_{tot} = E_1 + E_2 = E_0 \hat{e} e^{i(kx-\omega t)} \{(1+e^{i\phi})\} = 2\cos\left(\frac{\phi}{2}\right) e^{i\frac{\phi}{2}}$

$= 2 E_0 e^{i(kx-\omega t)} \hat{e} \cos\left(\frac{\phi}{2}\right) e^{i(kx-\omega t+\frac{\phi}{2})}$

Ex # & Energy Density

$E(x,t) = E_0 \hat{e} \cos(kx-\omega t)$ $\quad B(x,t) = \frac{E_0}{c} \hat{b} \cos(kx-\omega t)$

(a) Energy Density $\quad u = u_e + u_B = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{\mu_0 2} \Rightarrow \langle u \rangle = \langle u_e \rangle + \langle u_B \rangle$

(b) $u = n_\gamma E_\gamma \leftarrow E_\gamma = \hbar\omega$

* use $\langle E^2 \rangle = \frac{E_0^2}{2}$

anal. for B

Props of Conductors Ex

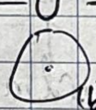
Setup Spherical conductor w/ radius R in $E_{ext} = E_0 \hat{z}$
surface grounded to $V=0$ @ surface

(a) Induced charge on surface $\quad \sigma(\theta) = \sigma_0 \cos\theta$ $\quad$ (b) potential const throughout

(c) F on conductor $= q E_{ext} = \int_0^\pi \sigma(\theta)\, dA = \int_0^\pi \sigma_0 \cos(\theta)(R^2 \cos\theta)\, d\theta$

$= \sigma_0 R^2 \left(-\frac{1}{2}\cos^2\theta\right)\Big|_0^\pi = \frac{\sigma_0 R^2}{2} \Rightarrow F = \frac{\sigma_0 R^2}{2} E_0$

Faraday Law Ex



$B(t) = B_0 \hat{z} \cos(\omega t)$ $\quad$ (a) $EMF = -\frac{d}{dt}\left(B_0 \cos(\omega t) \cdot \pi R^2\right)$

(b) $P(t) = I(t)^2 R_{loop}$ for $I(t) = \frac{\mathcal{E}}{R_{loop}}$ $\quad E = \int_0^T P(t)\, dt$

Closed Loop Mag Dist. Ex