non trivial nullspace $\iff$ det $= 0 \iff$ linearly dependent colums $\iff$ non invertible

trivial nullspace $\iff$ det $\neq 0 \iff$ linearly independent colums $\iff$ invertible

Linearity:

Homogeneity $f(ax) = af(x)$

Superposition $f(x_1 + x_2) = f(x_1) + f(x_2)$

affine: does not fix the origin

If a row is:

000 ... 10 $\Rightarrow$ infinite sols.

000 ... 1n $\Rightarrow$ no sols.

$n \in \mathbb{R}$ but not 0

$$\begin{array}{cc} & n \\ m & \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1 \begin{bmatrix} \vdots \end{bmatrix} + x_2 \begin{bmatrix} \vdots \end{bmatrix} + x_3 \begin{bmatrix} \vdots \end{bmatrix}$$

$$A\vec{x} = \vec{b} \Rightarrow A \in \mathbb{R}^{m \times n}, \vec{x} \in \mathbb{R}^n, \vec{b} \in \mathbb{R}^m$$

Conservative state transition matrix:
columns all sum to 1

If $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$ represents state,

$$A = \begin{bmatrix} A \to A & B \to A & C \to A \\ A \to B & B \to B & C \to B \\ A \to C & B \to C & C \to C \end{bmatrix}$$

Vector Spaces:
- must contain $\vec{0}$
- closed under vector addition
- closed under scalar multiplication

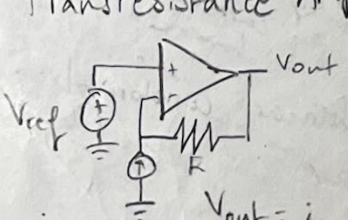
$\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots, \vec{v}_n\}$ is the basis for $\mathbb{V}$ (vector space)

if $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are lin. indep.
& for any $\vec{v} \in \mathbb{V}$, $\vec{v}$ is a linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$

dimension of $\mathbb{V}$ = num. of basis vectors

$A^T = a_{xy} \Rightarrow a_{yx}$

Transresistance Amplifier



$V_{out} = i_{in}(-R) + V_{ref}$

Row Echelon Form:
- all nonzero rows are above all zero rows
- leading entries of a nonzero row are always to the right of the leading entries of the row above it
- all leading entries of non zero rows = 1

RREF:
- in REF
- each leading entry of a nonzero row is the only nonzero entry in its column

★ variables corresponding to colums containing leading entries are called basic variables, all others are called free variables

$$\text{span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n) = \left\{ \sum_{i=1}^{n} a_i \vec{v}_i \mid a_i \in \mathbb{R} \right\}$$

$\text{span}(A) = \text{Col}(A) = \text{range}(A)$

$\text{span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n) = \mathbb{R}^n$ if $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly independent

$AA^{-1} = A^{-1}A = \mathbb{I}$

$[A \mid \mathbb{I}] \to [\mathbb{I} \mid A^{-1}]$
using Gauss. Elim

if A is invertible, there exists a unique solution to $A\vec{x} = \vec{b}$ for all $\vec{b}$

Vector subspace $\mathbb{U} \subset \mathbb{V}$ if it
- is closed under addition
- is closed under scalar multiplication
- contains $\vec{0}$

Column Space = span(A)
↳ dim = # of linearly indep. cols.

$\dim(\text{span}(A)) \leq \min(m, n)$

$\text{rank}(A) = \dim(\text{span}(A))$

$\text{Null}(A) = \{\vec{x} \mid A\vec{x} = \vec{0}, \vec{x} \in \mathbb{R}^m\}$
↳ $\vec{0}$ is trivial

Rank-Nullity Theorem:

$m - \dim(\text{span}(A)) = \dim(\text{Null}(A))$

$m - \text{rank} = \dim(\text{Null}(A))$

$\vec{v}$ is an eigenvector
$\vec{v} \neq \vec{0}$
$\vec{v}$ direction remains unchanged by A

$A\vec{v} = \lambda\vec{v}$

$\Rightarrow A\vec{v} - \lambda I \vec{v} = \vec{0}$

$(A - \lambda I)\vec{v} = \vec{0}$

$\det(A - \lambda I) = 0$

λ is an eigenvalue
↳ dictates scale factor of eigenvector

$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$

↳ factor by which area/volume changes as a result of transformation

eigenvectors corresponding to distinct eigenvalues are lin. indep.

let $\vec{x}$ be original vector
$\vec{x}'$ be transformed vector

$CBA\vec{x} = \vec{x}'$

$\longleftarrow$ order transformations are applied in

steady-state vector is eigenvector associated with $\lambda = 1$

$A \in \mathbb{R}^{m \times n}$

$B \in \mathbb{R}^{n \times p}$

$AB \in \mathbb{R}^{m \times p}$

Counter clockwise

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Electrical Circuit Analysis

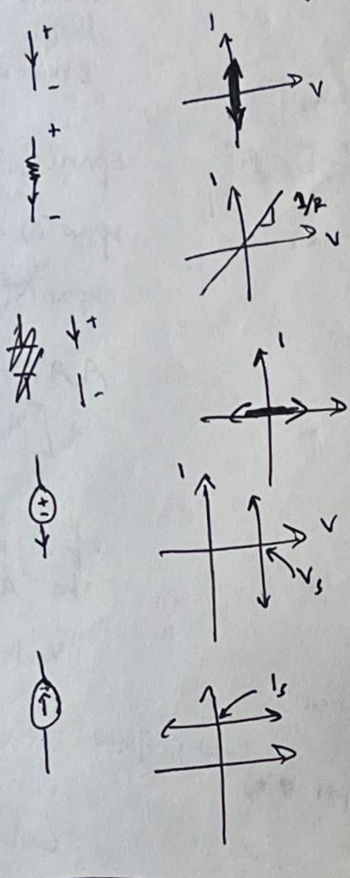
wire

resistor

open circuit

voltage source

current source

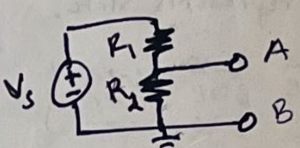


Ohm's Law $V = IR$

Kirchoff's Law

Current: $\sum_{node} i_k = 0$

Voltage: $\sum_{loop} v_k = 0$



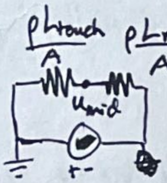
$V_{AB} = V_s\left(\frac{R_2}{R_1 + R_2}\right)$

$I = \frac{dQ}{dt}$

$R = \frac{\rho L}{A}$

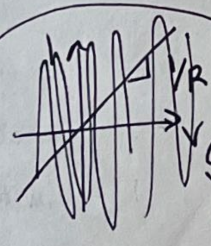
$P = IV = \frac{V^2}{R} = I^2 R$

Resistive Touchscreen



$u_{mid} = \frac{R_2}{R_1 + R_2} V_s = \frac{L_{touch}}{L} V_s$

one for horizontal position, one for vertical



Dependent Sources:

Voltage Controlled Voltage Source αV_i

gV_i Voltage Controlled Current Source

Current controlled Voltage Source rI_i

αI_i Current Controlled Current Source

can describe as a linear combination of independent sources

Superposition:

set all ~~voltage~~ independent voltage sources to 0 (~~open~~ wire) ← 0 resistance

set all current sources to 0 (open circuit) ← ∞ resistance

calculate current/voltage of dependent element

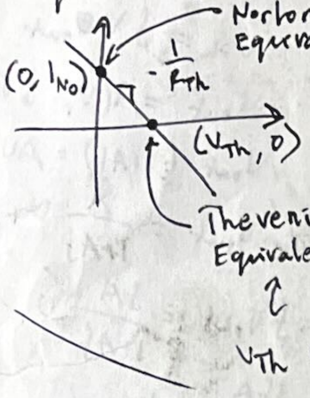
add all values

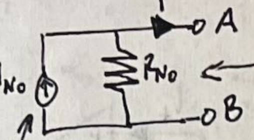
Voltmeter: ∞ resistance in parallel

Ammeter: 0 resistance in series

$$A \| B = \frac{AB}{A+B}$$

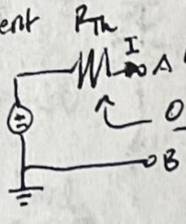
Equivalence:





short circuit across terminals, current through = I_{No}

$R_{No} = R_{Th}$



$V_{AB} = V_{Th}$

0 any independent sources, apply I_{test} to terminals and measure V, or apply V_{Test} and measure I

$R = \frac{V}{I}$

$$Q = CV_c$$

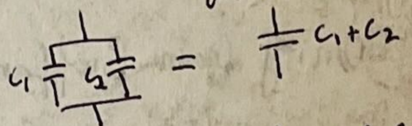
$$\Rightarrow \frac{dQ}{dt} = C\frac{dV_c}{dt} = I$$

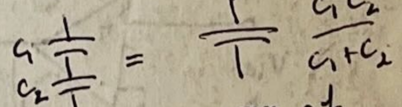
$$\Rightarrow \int_0^t I\,dt = C\int_0^{V_c} dV_c$$

(assume I is constant over 0-t

$$\Rightarrow V_c(t) = \frac{I}{C}t + V_c(0)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_c}{dt}}$$





$$C = \frac{\varepsilon A}{d}$$ ← area of overlap

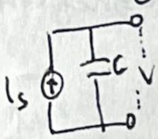
$$V_c(t) = \frac{I}{C}(t - t_0) + V_c(t_0)$$

$dE = V_c\,dq$

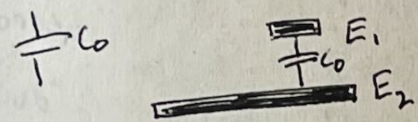
$dq = C\,dV_c$

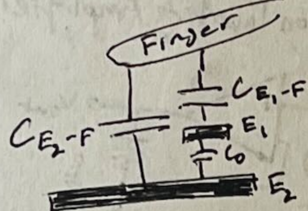
$E = \frac{CV^2}{2}$ ← voltage applied across capacitor

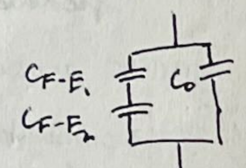
$V = \frac{I_s t}{C}$



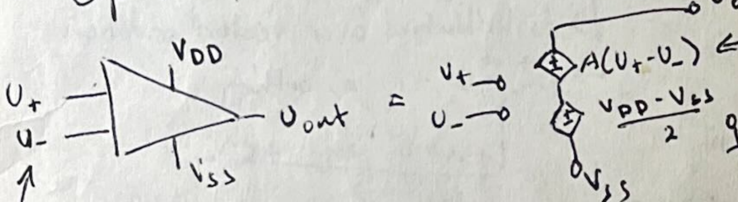
Capacitive Touchscreen





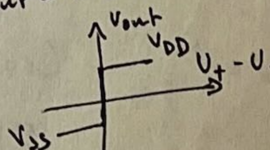


Op Amp



$V_{SS} \le U_{out} \le V_{DD}$

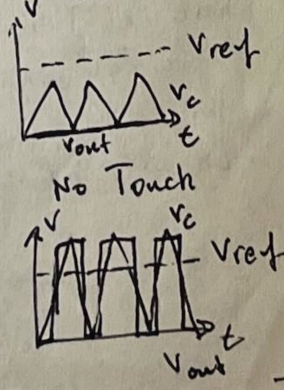
$$U_{out} = A(U_+ - U_-) + \frac{V_{DD}-V_{SS}}{2}$$



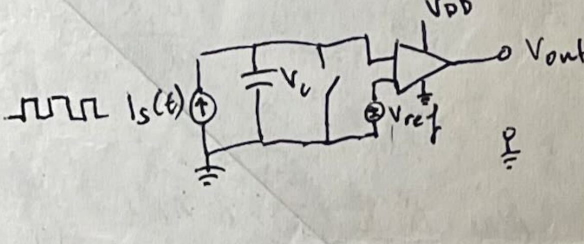
= Vref to use as comparator

V_{DD} if $U_+ > V_{ref}$

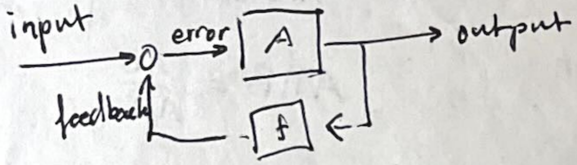
V_{SS} if $U_+ < V_{ref}$



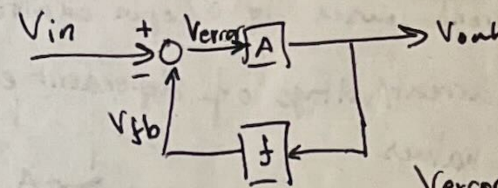
if no Vout, finger is present



~~Negative~~ Feedback



Negative Feedback



$V_{error} = U_+ - U_-$

$V_{out} = A V_{error} = A(U_+ - U_-)$

$U_- = f V_{out}$

$V_{out} = A(U_+ - f V_{out})$

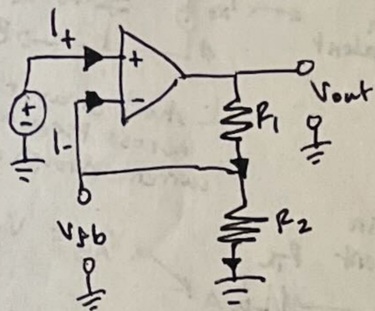
$V_{out}(1 + Af) = AU_+$

$V_{out} = \frac{A}{1+Af} U_+$

$U_- = f V_{out} = \frac{fA}{1+Af} U_+$

$\lim_{A \to \infty} \frac{fA}{1+Af} = 1$

$\Rightarrow U_+ = U_-$



Golden Rules:

$I_- = I_+ = 0$ ← always true

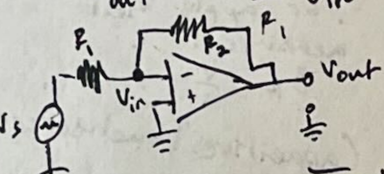
$u_+ = u_-$ ← error signal going into op amp = 0

$A \to \infty$ implies

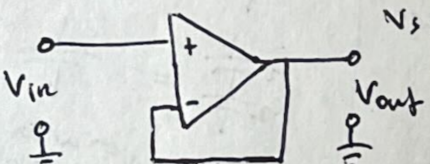
only when negative feedback

Inverting Amplifier

$V_{out} = -\frac{R_2}{R_1} V_{in}$



Unity Gain Buffer

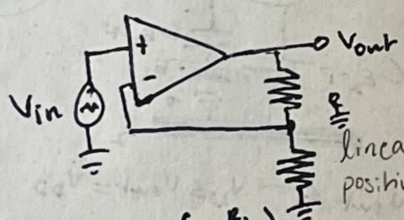


$V_{in} = V_{out}$

Test Negative feedback:

- zero out all independent sources
- "dink" V_{out}, if feedback is in opposite direction $\Rightarrow$ negative feedback

Non Inverting Amplifier



$V_{out} = (1 + \frac{R_1}{R_2}) V_{in}$

Net Power in closed circuit = 0

Inner product $\langle \vec{x}, \vec{y} \rangle = \sum_{i=1}^{n} x_i y_i = \vec{x}^T \vec{y}$

linear, symmetry → commutative

positive definite → scalar multiplication

→ distributive over vector addition

→ $\langle \vec{x}, \vec{y} \rangle = 0 \Rightarrow$ orthog

Euclidian Norm

$\sqrt{\langle \vec{x}, \vec{x} \rangle}$

→ $\|\vec{x}\| \geq 0$

→ $\|\vec{x}\| = 0$ iff $\vec{x} = \vec{0}$

→ $\|\alpha \vec{x}\| = |\alpha| \|\vec{x}\|$

→ $\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$ (Triangle Inequality)

→ $\langle \vec{x}, \vec{y} \rangle = \cos \alpha$ if α is angle between unit vectors $\vec{x}$ and $\vec{y}$

→ $|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \|\vec{y}\|$ (Cauchy - Schwartz Inequality)

Cross Correlation

$corr_{\vec{x}} \vec{y}[k] = \sum_{i=-\infty}^{\infty} x[i] y[i-k]$

where $x[i], y[i] = 0$ for any i $\vec{x}, \vec{y}$ are not defined for

↳ $corr_{\vec{x}} \vec{y} \neq corr_{\vec{y}} \vec{x}$

Least Squares

$A\vec{x} \approx \vec{b}$ $\quad \|\vec{e}\| \perp span(A)$

projection of $\vec{b}$ onto span(A) minimizes $\|e\|$ $= \|b - A\vec{x}\|$

$proj_{\vec{a}_1} \vec{b} = \frac{\langle \vec{b}, \vec{a}_1 \rangle}{\langle \vec{a}_1, \vec{a}_1 \rangle} \vec{a}$

$\vec{x} = (A^T A)^{-1} A^T \vec{b}$

$proj_{span(A)} \vec{b} = \sum_{i=1}^{k} \frac{\langle b, a_i \rangle}{\langle a_i, a_i \rangle} a_i$ where a_i is a column of A

$A^T(\vec{b} - A\vec{x}) = \vec{0} \Rightarrow A^T A \vec{x} = A^T \vec{b}$

$$\begin{bmatrix} 2(\vec{a}_1 - \vec{a}_2)^T \\ 2(\vec{a}_1 - \vec{a}_3)^T \end{bmatrix} \vec{x} = \begin{bmatrix} \|\vec{a}_1\|^2 - \|a_2\|^2 - d_1^2 + d_2^2 \\ \|\vec{a}_1\|^2 - \|a_3\|^2 - d_1^2 + d_3^2 \end{bmatrix}$$

$\vec{a}$ is coordinates of sender

d is distance from sender to reciever

Left Nullspace

$\{\vec{x} : A^T\vec{x} = \vec{0}\}$

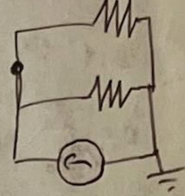
let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\frac{1}{|M|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

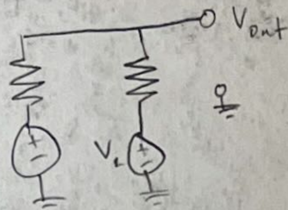
If matrix has more columns than rows (more unknowns than eqs), a unique solution does not exist

Current Divider:

$$I_{R_1} = \frac{I_s R_2}{R_1 + R_2}$$

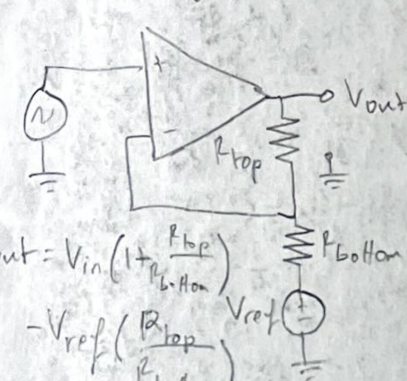


Voltage Summer



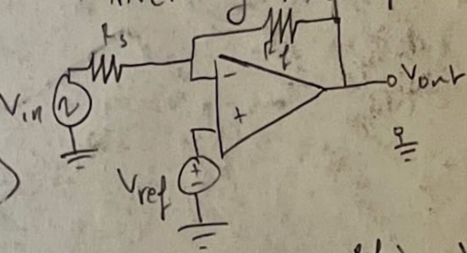
$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right) + V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Non-Inverting Amplifier



$$V_{out} = V_{in}\left(1 + \frac{R_{top}}{R_{bottom}}\right) - V_{ref}\left(\frac{R_{top}}{R_{bottom}}\right)$$

Inverting Amplifier

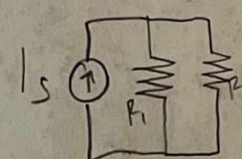


$$V_{out} = V_{in}\left(-\frac{R_f}{R_s}\right) + V_{ref}\left(\frac{R_f}{R_s} + 1\right)$$

$\langle \vec{v}, \vec{w} \rangle = \vec{v}^T Q \vec{w}$

Q must be symmetric

$Q = Q^T$



$$I_{R_1} = \frac{R_2}{R_1 + R_2} I_s$$

Current Divider

$\text{null}(A) = \text{null}(A^T A)$